The Enigma of False Bias Detection in a Strapdown System During Transfer Alignment

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This work describes a phenomenon discovered during in-flight transfer alignment of a strapdown inertial navigation system. The phenomenon, which has not been reported in the literature before, is that of false longitudinal accelerometer bias estimation by the Kalman filter employed in the transfer alignment. Reference data timing error is suggested as the source of this error and theoretically analyzed. From the analytic treatment it is obvious that false longitudinal accelerometer bias estimation can occur only in strapdown and not in gimbaled inertial navigation systems. True covariance simulation is performed which reinforces the analytic explanation and yields further insight into the false bias generation mechanism. A final validation of the explanation is obtained when the proposed cause for the phenomenon is removed, eliminating the false bias estimate.

I. Introduction

OFTEN an inertial navigation system (INS) has to be aligned in-flight. In these cases it is necessary to obtain some reference information on the state of the aligned INS (slave) such as its position, its velocity or its angular rate. Usually the reference information is velocity which is supplied by another, reference, INS. In the alignment stage the velocity computed by the reference INS (master) is compared with that computed by the slave INS and the difference, which is indicative of the slave misalignment with respect to the master, is processed by a Kalman filter (KF) which yields misalignment data as well as the slave gyro and accelerometer error data. The latter data are known as calibration data and its removal from the respective gyro and accelerometer outputs is known as calibration. The operation of aligning a slave INS with a master INS comparing quantities computed by both INS is known as transfer alignment (TA). While the estimation of the level misalignment is accomplished rather quickly, the azimuth misalignment is time consuming unless the delivery vehicle performs alignment maneuvers which enhance the observability of the azimuth misalignment error.[1-9] These maneuvers enable the TA Kalman filter to separate between the tilt errors and the accelerometer biases which othewise balance each other and, to a certain degree, are unobservable. Following the developments in gyro and computer technology which took place in the last twenty years, the trend today is to use strapdown INS (SDINS) as the slave navigation system. This choice reflects the savings in weight, volume, complexity, and cost achieved when SDINS, rather than gimbaled INS, is used.

On the system level, SDINS usually behave exactly like gimbaled INS. There are, however, some phenomena which are characteristic to SDINS only. One such phenomenon is the destruction, with heading change, of the balance between the accelerometer bias and the tilt error achieved during self leveling of INS[10-12]. *The present paper treats another phenomenon which takes place in SDINS only*—false accelerometer bias detection by the KF.

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Section II presents the discovery of the phenomenon at a test. In Sec. III a solution is suggested to the perplexity presented in Sec. II. An analytic explanation is also provided in that section. This explanation is then verified in Sec. IV through true covariance analysis. The explanation is further validated by manipulation of the test results. This is presented in Sec. V. The conclusions are discussed in Sec. VI.

II. Phenomenon Description

In order to describe the discovery of the phenomenon we first have to outline the TA experiments which were performed when the phenomenon was noticed. A master and a slave INS were installed on a light test aircraft. The master INS was updated using external position information such that for all practical purposes its indicated velocity could be considered error free. The slave system consisted of a strapped-down inertial measurement unit (IMU) which measured the vehicle angular rate and specific force vectors. The master-computed position and velocity data were recorded on a tape. Similarly the strapdown IMU-measured angular rate and specific force vectors were also recorded on the tape. The latter data were read off-line in the laboratory and processed to yield SDINS position and velocity information. The computed slave SDINS velocity was compared with the master velocity which was read off the tape. The difference between the two was processed by a TA KF algorithm to yield estimates of the SDINS attitude errors as well as its IMU accelerometer random constant errors.

The baseline flight trajectory during the experiments consisted of four segments. The nature and purpose of each segment are listed in Table 1. The transfer alignment was performed during the first three segments of the trajectory. During the first segment the estimation of most of the tilt errors is achieved. The second segment has two purposes. First, the lateral acceleration serves to enhance azimuth observability and estimation,[7] and second, it serves to separate the accelerometer biases from the residual tilt errors and thus enable the KF to estimate these biases. The third segment is inserted into the baseline trajectory in order to perform further estimation of the tilt errors. These three segments constitute the whole TA trajectory. In order to examine the effectiveness of the TA, a fourth segment was added on which the SIDNS IMU was navigating indepen-

Table 1 Baseline flight trajectory

Mode	Segment no.	Nature of trajectory	Purpose	Off-line computation
	1	Straight and level flight	Initial leveling	Navigation, TA KF
TA	2	S-shaped	Azimuth alignment	Navigation, TA KF
	3	Straight and level flight	Final leveling	Navigation, TA KF
SDINS navigation	4	Straight and level flight	Simulation of SDINS navigation after TA	Navigation

dently based on the preceding alignment and calibration results.

The test aircraft performed two TA and SDINS-navigation flights repeating the trajectory described in Table 1. However, while in the first experiment the heading of segments 1, 3, and 4 were all the same, in the second experiment the heading of segment 4 differed by 20 deg from the heading of segment 3.

When the test data were read off the tape and processed in the lab, it was noticed that in both experiments the estimated x axis accelerometer bias exceeded its anticipated value by far. (The x axis of the IMU pointed along the longitudinal axis of the aircraft.)

To demonstrate the difference between the computed and expected x axis accelerometer bias estimates we turn to Fig. 1. The shaded area is bounded by the anticipated extremal values of the bias. We expect the estimated bias to fall within this region. The computed estimate, however, was entirely off and, as seen from curve a, even reached 5 times the extremal value.

In addition to the exaggerated bias estimate, it was also observed that in the second experiment, the SDINS-computed position during the simulated SDINS navigation (on the fourth segment) contained a large and unexpected error.

The latter observation is explained quite easily as the result of the erroneous estimate of the x axis accelerometer bias. Accordingly, since due to the incorrect estimate an incorrect value was used to compensate the reading of the x axis accelerometer bias, the position computation yielded an error. In fact, it was found that the unexpected part of the position error was compatible with the unexpected part of the accelerometer bias estimate. The reason why the exaggerated position error was found only in the second experiment is explained by the 20 deg difference between the third and fourth segment headings. On the third segment of the TA trajectory the incorrect accelerometer bias estimate generated an incorrect tilt error, each of which compensated the other. Thus, since the SDINS navigation was simulated on a straight and level flight whose heading was that of the third segment, the incorrect accelerometer bias estimate was still compensated by an incorrect tilt error, therefore the bias error did not manifest itself in an erroneous position computation. In the second experiment, though, the aircraft changed heading just after the SDINS started navigating, practically destroying tilt compensation of the erroneous bias estimate,[10-12] which caused a large position error build-up.

III. Analytic Explanation

In this experiment two different systems (master and SDINS) with independent clocks had to be synchronized in some fashion. (This problem is frequently encountered by engineers who try to mate two self-contained digital systems which were not initially designed to operate together). It was, therefore, suggested that *a time delay between the data generated by the master INS and that generated by the SDINS IMU* caused the unexpected large bias estimate.

In this section it will be shown that time delay does indeed generate such a bias error. In order to isolate the phenomenon from other error-generating factors, let us assume that the IMU of a SDINS which undergoes TA is error-free. Let us also assume that the master navigation system is error-free also. The translatory error propagation equation of the SDINS is[13]

$$\dot{\bar{v}} + (2\bar{\rho} + \bar{\Omega}) \times \bar{v} = \bar{\nabla} - \bar{\psi} \times \bar{f} \tag{1}$$

where $\bar{v}$ is the velocity error vector, $\bar{\rho}$ the angular rate vector of the local-level north-pointing coordinate system with respect to Earth corresponding to the SDINS position, $\bar{\Omega}$ the Earth rate vector, $\bar{\nabla}$ the accelerometer bias vector of the three accelerometer axes, $\bar{\psi}$ the angular misalignment vector between the "platform" and the computer coordinate systems,[4] and $\bar{f}$ the specific force vector. Due to the short duration of the TA stage, Eq. (1) can be simplified. Thus a simplified version of Eq. (1) was developed, in the state space, and implemented in our TAKF. In particular, the equations for the propagation of the three velocity error components were

$$\begin{bmatrix} \dot{v}_N \\ \dot{v}_E \\ \dot{v}_D \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & g & f_N & \cos A & -\sin A\cdot\cos\phi & \sin A\cdot\sin\phi \\ 0 & 0 & 0 & -g & 0 & -f_E & \sin A & \cos A\cdot\cos\phi & -\cos A\cdot\sin\phi \\ 0 & 0 & 0 & -f_N & f_E & 0 & 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} v_N \\ v_E \\ v_D \\ \psi_N \\ \psi_E \\ \psi_D \\ \nabla_x \\ \nabla_y \\ \nabla_z \end{bmatrix} \tag{2}$$

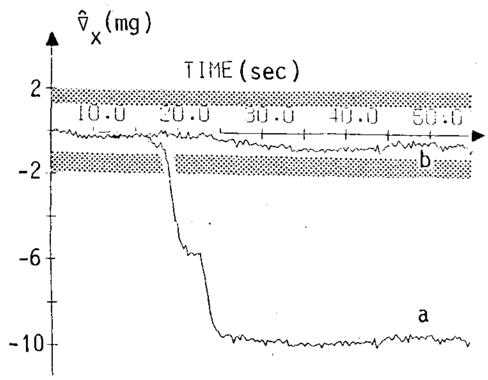

Fig. 1 Estimate of x-accelerometer bias: a) When the TA KF operated on the raw data read off the tape; b) When the TA KF operated on the corrected data.

where A is the heading angle and ϕ the aircraft roll angle.

Note that the state vector included also the gyro constant drifts, however, they do not influence $\dot{v}_N$, $\dot{v}_E$ and $\dot{v}_D$ directly and that is why they do not appear in Eq. (2). (The model also included accelerometer white noise components, but they play no role in the generation of the erroneous bias.) Since the aircraft velocity V_0 is constant during the TA maneuver, we can write

$$f_N = -V_0\dot{A}\cdot\sin A \tag{3a}$$

$$f_E = V_0\dot{A}\cdot\cos A \tag{3b}$$

For the ensuring development it is convenient to use Eqs. (3) and present Eq. (2) in the following form:

$$\dot{v} = \begin{bmatrix} 0 \\ -g \\ -V_0\dot{A}\cdot\cos A \end{bmatrix}\psi_N + \begin{bmatrix} g \\ 0 \\ -V_0\dot{A}\cdot\sin A \end{bmatrix}\psi_E + \begin{bmatrix} V_0\dot{A}\cdot\cos A \\ V_0\dot{A}\cdot\sin A \\ 0 \end{bmatrix}\psi_D + \begin{bmatrix} \cos A \\ \sin A \\ 0 \end{bmatrix}\nabla_x + \begin{bmatrix} -\sin A\cdot\cos\phi \\ \cos A\cdot\cos\phi \\ \sin\phi \end{bmatrix}\nabla_y + \begin{bmatrix} \sin A\cdot\sin\phi \\ -\cos A\cdot\sin\phi \\ \cos\phi \end{bmatrix}\nabla_z \tag{4}$$

where $f^T = [\dot{v}_N, \dot{v}_E, \dot{v}_D]$. The observable of the TA KF is v; that is, the KF is designed to interpret the data fed into it as a measurement of v. Let us examine, then, what is really fed as data when there is a time delay between the two navigation systems. That data, which we denote by z, is the difference between the SDINS-computed velocity and the reference velocity of the master; that is

$$z = V_{\text{SDINS}} - V_{\text{ref}} \tag{5}$$

and

$$V_{\text{SDINS}} = V(t) + v \tag{6}$$

where $V(t)$ is the true velocity of the aircraft. Due to the time delay, Δ, between the master and slave systems, and since we assumed that the master velocity was practically error free, V_{ref} is the true velocity delayed by Δ; that is

$$V_{\text{ref}} = V(t-\Delta) \tag{7}$$

A first-order Taylor series expansion of the right-hand side of Eq. (7) yields

$$V_{\text{ref}} = V(t) - \dot{V}(t)\Delta \tag{8}$$

Substitution of Eqs. (6) and (8) into Eq. (5) yields

$$z = v + \dot{V}(t)\Delta \tag{9}$$

The aircraft velocity vector is

$$V(t) = \begin{bmatrix} V_0\cdot\cos A \\ V_0\cdot\sin A \\ 0 \end{bmatrix} \tag{10}$$

therefore,

$$\dot{V}(t) = V_0\dot{A}\begin{bmatrix} -\sin A \\ \cos A \\ 0 \end{bmatrix} \tag{11}$$

Substitution of Eq. (11) into Eq. (9) then yields

$$z = v + V_0\dot{A}\begin{bmatrix} -\sin A \\ \cos A \\ 0 \end{bmatrix}\Delta \tag{12}$$

In order to relate the data z to $\dot{v}$ of Eq. (4) we differentiate Eq. (12) with respect to time and obtain

$$\dot{z} = \dot{v} - V_0\dot{A}^2\begin{bmatrix} \cos A \\ \sin A \\ 0 \end{bmatrix}\Delta + V_0\ddot{A}\begin{bmatrix} -\sin A \\ \cos A \\ 0 \end{bmatrix}\Delta \tag{13a}$$

along circular segments of the TA maneuver $\dot{A}$ is constant and $\ddot{A}$ vanishes. Then

$$\dot{z} = \dot{v} - V_0K^2\begin{bmatrix} \cos A \\ \sin A \\ 0 \end{bmatrix}\Delta \tag{13b}$$

where K is the constant value of $\dot{A}$. In our search for the source of the phenomenon, we were looking for an unmodeled error generating mechanism which introduced into the system an error that behaved like the error generated by ∇_x. Examination of Eq. (13b) reveals that the constant time delay, Δ, is multiplied by a mode which is identical to the mode which multiplies the constant bias, ∇_x, in Eq. (4). This was the reason why it was hypothesized that the time delay was the cause for the excessive estimate of ∇_x. Now since the mode which ∇_x is capable of introducing into the derivative of the SDINS velocity error does exist in the derivative of the observable, the KF interprets the existence of this mode in the data it obtains as a contribution due to ∇_x. In other words, the measured information carries a signature which is characteristic of the signature that ∇_x bears on the observable. Thus the KF, which is "unaware" of the delay, attributes the observed error to an existence of ∇_x.

It is interesting to note that although the TA maneuver is averaged such that the final velocity vector is equal to the initial one, the influence of the delay is not averaged but rather rectified. This is seen in Eq. (13b) where $\dot{A}$ is raised to the second power; that is, the influence of the time delay is always in the same direction whether the lateral acceleration (or $\dot{A}$) is positive or negative. Observe in Eq. (4) that ψ_D seemingly bears signature which is also similar to that of the time delay Δ. There is however a small but very important difference between the two; namely, ψ_D is multiplied by $\dot{A}$ and not by $\dot{A}^2$; therefore the rectification effect does not exist there. For this reason the KF attributes the part of $\dot{z}$ which is caused by the time delay to ∇_x and not to ψ_D [see Eq. (4)].

Recall that in the preceding explanation it was argued that on circular segments $\dot{A}$ was constant and thus $\ddot{A}$ vanished. Under these conditions the signature of the time delay, Δ, is identical to that of ∇_x. In reality these conditions are not entirely met. Yet as proven, through both covariance analysis and experiment (see next sections), the phenomenon is still very prominent. This stems from the fact that even when the observable does not perfectly match the dynamic model in the KF, of all states modeled in the filter, the behavior of ∇_x matches the behavior of the observable best.

Note that one can easily be led to believe that a small time delay on the order of magnitude of tens of milliseconds is unnoticeable. The argument is as follows: the nominal velocity vector $V(t)$ may change only very little during a couple of hundreths of a second, let alone the SDINS velocity error $v(t)$, which during this time range stays practically constant. This argument is false since, as seen in Eq. (9), it is not the change of $v(t)$ that matters but rather the size of the term $\dot{V}(t)\Delta$ which, depending on the acceleration vector, $\dot{V}(t)$, may reach significant values. At any rate the values are significant enough to cause the false bias estimates.

IV. True Covariance Simulation

A further confidence in the preceding explanation of the phenomenon is gained when the model which is claimed to have generated the false bias is used in a true covariance simulation and the latter does indeed generate the anticipated standard deviation of ∇_x. Furthermore, the covariance simulation can also be used to gain better insight into the various details of the bias generation mechanism. By true covariance simulation we mean a covariance simulation which employs both the "truth model" and the "filter model" in a correct manner[15,16] and does not assume, a priori, that the covariance matrix which generates the Kalman gain is the true covariance matrix.

Truth Model

As shown in Eq. (9), due to the time delay, the observable, z, is not just the SDINS computed velocity error, v, but rather

$$z = v + \dot{V}(t)\Delta \tag{14}$$

Adding a white noise measurement which is usually involved in the observation process and resolving it into its north, east, and down components, Eq. (14) can be written as

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} v_N \\ v_E \\ v_D \end{bmatrix} + \begin{bmatrix} \dot{V}_N \\ \dot{V}_E \\ \dot{V}_D \end{bmatrix} \Delta + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} \tag{15}$$

where n_1, n_2 and n_3 are the three white noise measurement components whose covariance matrix at time point t_K is known and is denoted by R_K. $\dot{V}_N$, $\dot{V}_E$ and $\dot{V}_D$ are the time derivatives of the north, east, and down components of the aircraft nominal velocity.

The SDINS-computed velocity error components, v_N, v_E and v_D, are states in the state vector of the model describing the propagation of errors in the SDINS. The time delay Δ is assumed to be a zero-mean constant random variable whose dynamics equation is of course

$$\dot{\Delta} = 0 \tag{16}$$

As a standard procedure[17] Eq. (16) has to be augmented with the SDINS error propagation equation. The final model in

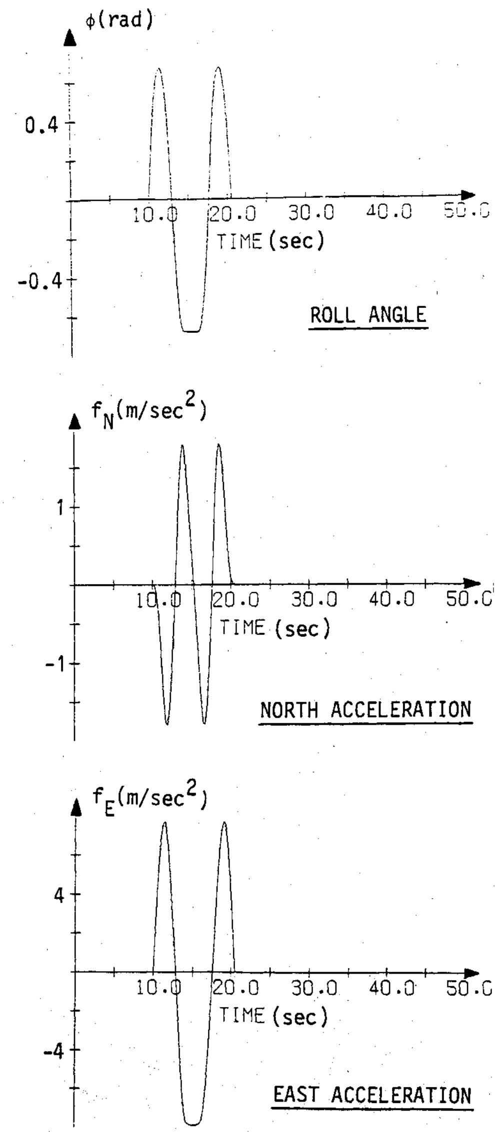

Fig. 2 Baseline TA maneuver for the true covariance simulation runs.

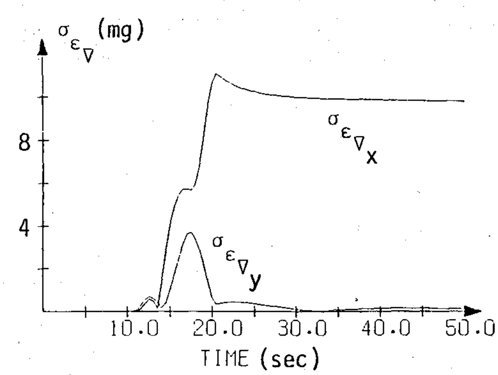

Fig. 3 Standard deviation of the estimation error of the x and y accelerometer bias.

our case is as follows

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} r \\ v \\ \psi \\ \nabla \\ \epsilon \\ \Delta \end{bmatrix} = \left[\begin{array}{c:c:c:c} F_9 & \begin{array}{c} 0 \\ D_L^b \\ 0 \end{array} & \begin{array}{c} 0 \\ 0 \\ D_L^b w \end{array} & 0 \\ \hdashline 0 & & 0 & \end{array}\right]\begin{bmatrix} r \\ v \\ \psi \\ \nabla \\ \epsilon \\ \Delta \end{bmatrix} + \left[\begin{array}{c} 0 \\ \hdashline D_L^b{}_\nabla \\ \hdashline D_L^b w_\epsilon \\ \hdashline 0 \end{array}\right] \tag{17}$$

where r, v, and ψ are respectively the position, velocity, and attitude error vectors resolved in the local-level north-pointing (L) coordinate system. ∇ and ϵ are the accelerometer bias and gyro constant drift rate vectors given in the body b coordinate system. F_9 is the standard 9×9 INS error dynamics matrix,[13] D_L^b is the transformation matrix from the b to the L coordinate system and, finally, w_∇ and w_ϵ are the accelerometer and the gyro white noise components given in the body axes. The observation matrix H which corresponds to both Eqs. (15) and (17) is

$$H = \left[\begin{array}{ccc:ccc:c:c} 0 & 0 & 0 & 1 & & & 0 \text{ --- } 0 & \dot{V}_N \\ 0 & 0 & 0 & & 1 & & 0 \text{ --- } 0 & \dot{V}_E \\ 0 & 0 & 0 & & & 1 & 0 \text{ --- } 0 & \dot{V}_D \end{array}\right] \tag{18}$$

Design Model

Basically the design model is the model implemented in the filter which, of course, does not account for the time delay. Since we have determined that the gyro bias does not participate in the mechanism which generates the false bias estimate, we did not include it in the design model. Consequently, the design model is as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} v_N^* \\ v_E^* \\ v_D^* \\ \psi_N^* \\ \psi_E^* \\ \psi_D^* \\ \nabla_x^* \\ \nabla_y^* \\ \nabla_z^* \end{bmatrix} = \left[\begin{array}{ccccccccc} 0 & 0 & 0 & 0 & g & f_N & \cos A & -\sin A\cdot\cos\phi & \sin A\cdot\sin\phi \\ 0 & 0 & 0 & -g & 0 & -f_E & \sin A & \cos A\cdot\cos\phi & -\cos A\cdot\sin\phi \\ 0 & 0 & 0 & -f_N & f_E & 0 & 0 & \sin\phi & \cos\phi \\ \hdashline & & & & 0 & & & & \end{array}\right]\begin{bmatrix} v_N^* \\ v_E^* \\ v_D^* \\ \psi_N^* \\ \psi_E^* \\ \psi_D^* \\ \nabla_x^* \\ \nabla_y^* \\ \nabla_z^* \end{bmatrix} + \begin{bmatrix} 0 \\ D_L^b w_\nabla^* \\ \\ 0 \\ \\ \end{bmatrix} \tag{19}$$

and the observation matrix is simply

$$H^* = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \text{ ------ } 0 \\ 0 & 1 & 0 & 0 \text{ ------ } 0 \\ 0 & 0 & 1 & 0 \text{ ------ } 0 \end{array}\right] \tag{20}$$

Results

Figure 2 presents the baseline TA maneuver for which true covariance simulation was run. The velocity of the light test plane was 40 m/s and the maximum lateral acceleration was 7.85 m/s.[2] Figure 3 shows the variance of the true error in estimating ∇_x when the standard deviation of the time delay was 50 msec. In order to isolate the influence of the time delay, the truth model of the slave system represented an ideal SDINS with perfect measurements of its velocity components. The only error which the truth model accounted for was the time delay between its velocity outputs and the velocity outputs of the perfect master navigation system. It is interesting to compare the standard deviation of the x-accelerometer bias estimation error to that of the y-accelerometer. The latter is also shown in Fig. 3.

As was mentioned in Sec. III, although the mode multiplying ψ_D in Eq. (4) is similar to that inserted into the measurements by the time delay, it is multiplied by $\dot{A}$ which changes sign during the maneuver; therefore the standard deviation of the estimation errors of ψ_D is expected to oscillate and not grow to an appreciable value. This is indeed the case as demonstrated in Fig. 4, where unlike $\sigma_{\epsilon_{\nabla_x}}$, $\sigma_{\epsilon_{\psi_D}}$ oscillates and reaches a relatively low value. Also note that the peaks of $\sigma_{\epsilon_{\psi_D}}$ occur at times in which the growth of $\sigma_{\epsilon_{\nabla_x}}$ is checked. This stems from the fact that the KF assigns some of the measurement caused by the time delay to ψ_D as well, since at these points its mode also resembles the mode generated by the delay.

V. Validation Through Data Reprocessing

As a final and conclusive validation of the hypothesis, the data recorded from the SDINS IMU was delayed by 50 msec before it was processed by the navigation and TA KF routines such that the artificial shift now negated the initial sychronization error. Indeed, the estimate of ∇_x went down and remained within its anticipated range as shown by curve b of Fig. 1.

As a consequence of the analysis which was presented in this paper the equipment involved in the experiment was checked and the cause of the delay was identified and removed. The flight tests were then repeated and indeed the false bias estimate did not recur.

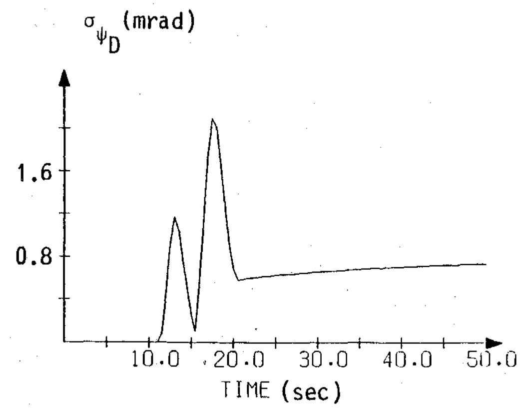

Fig. 4 Standard deviation of the estimation error of ψ_D

VI. Conclusions

The paper described a phenomenon which occurs only in strapdown INS. It happens during in-flight alignment and manifests itself in incorrect longitudinal accelerometer bias estimation which may cause large position errors. It was shown that time delay introduced into the measured data a signal which resembled the contribution which only a longitudinal accelerometer bias could have made. Therefore, the KF algorithm whose model is based on a dynamic system that contains the influence of that bias but not the influence of the time delay, identifies the signature of the time delay in the measured signal as the signature of the longitudinal accelerometer bias. Consequently, the KF produces an excessive estimate of the later.

It was shown that it is not the change in the slave velocity error (which is indeed rather small) but rather the level of the nominal acceleration existing during the maneuver which introduces the incorrect data. The latter may cause a considerable accelerometer bias and possibly position errors. It was shown that although the components of this acceleration change sign during the transfer alignment maneuver, their influence is not averaged out but is rather rectified.

An artificial cancellation of the time delay between the master and slave system data eliminated the incorrect bias estimate. Elimination of the cause for the time delay in the hardware resulted in good performance of the Kalman filter algorithm in succeeding flight tests.

The practical conclusion of this work is, then, that utmost care should be given to the synchronization of the two navigation systems in in-flight transfer alignment.

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